K. Thus, the high heating part of the entry is over before the expanded scale height associated witi the thermal bulge can alleviate entry heating.

Conclusions

Although Kliore's occultation measurements of Jupiter's atmosphere disagree with Earth-based observations and with another Pioneer 10 experiment, the impact on entry heating of Kliore's model of Jupiter's atmosphere was examined. It was found that the warm temperature bulge exists at a level too low in the atmosphere to affect entry heating and that the nominal atmosphere fits Kliore's model atmosphere best insofar as heating is concerned. Therefore, previous estimates of the heating levels to be expected for a probe entering Jupiter's atmosphere are not affected by Kliore's postulated atmospheres.

References

[1]Kliore, A., Cain, D. L., Fjeldbo, G., Seidel, B. L., and Rasool, S. I., "Preliminary Results on the Atmospheres of Io and Jupiter from the Pioneer 10 S-Band Occultation Experiment," *Science,* Vol. 183, Jan. 1974, pp. 323-324.

[2]Kliore, A., Cain, D. L., Fjeldbo, G., Seidel, B. L., and Rasool, S. I., "The Atmopsheres of Io and Jupiter Measured by the Pioneer 10 Radio Occultation Experiment," Preprint No. II-VII.1.4, 17th Plenary Meeting of COSPAR, Sao Paulo, Brazil, June 1974.

[3]Gulkis, S. and Paynter, R., 'Thermal Radio Emission from Jupiter and Saturn," *Physics of the Earth and Planetary Interiors,* Vol. 6, 1972, pp. 36-43.

[4]Houck, J., Pollack, J., Schaack, D., Reed, R., and Summers, A., "Jupiter: Its Infrared Spectrum from 16 to 40 Micrometers," *Science,* Vol. 189, Aug. 1975, pp. 720-722.

[5]Ohring, G., "The Temperature and Ammonia Profiles in the Jovian Atmosphere from Inversion of the Jovian Emission Spectrum," *Astrophysical Journal,* Vol. 184, 1973, pp. 1027-1040.

[6]Chase, S. C., Ruiz, R. D., Munch, G., Neugehauer, G., Schroeder, M., and Trafton, L. M., "Pioneer 10 Infrared Radiometer Experiment: Preliminary Results," *Science,* Vol. 183, Jan. 1974, pp. 315-317.

[7]"The Planet Jupiter (1970), NASA SP-8069, 1971.

[8]Tauber, M. E., "Atmospheric Entry Into Jupiter," *Journal of Spacecraft and Rockets,* Vol. 6, Oct. 1069, pp. 1103-1109.

[9]Tauber, M. E. and Wakefield, R. M., "Heating Environment and Protection During Jupiter Entry," *Journal of Spacecraft and Rockets,* Vol. 8, June 1971, pp. 630-636.

[10]Allen, J. J. and Eggers, A. J., "A Study of the Motion and Aerodynamic Heating of Ballistic Missiles Entering the Earth's Atmosphere at High Supersonic Speeds," Rept. 1381, 1958, NACA.

[11]Wilson, K. H., Woodward, H., Tauber, M., and Page, W., "Jupiter Probe Heating Rates," presented at the Symposium on Hypervelocity Radiating Flow Fields for Planetary Entry, Jet Propulsion Lab., Pasadena, Calif. 1972.

[12]Myers, H., private communication, Aug. 1974, McDonnell Douglas, St. Louis, Mo.

[13]Tauber, M. E., "Some Simple Scaling Relations for Heating of Ballistic Entry Bodies," *Journal of Spacecraft and Rockets,* Vol. 7, July 1970, pp. 885-886.

Stability Conditions for Spin-Stabilized Rockets

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Introduction

THE Hurwitz problem on the location of zeros of polynomials in the complex plane finds practical application in analyzing questions of stability.[1] The purpose of the present study is to make use of the same criterion to derive the stability conditions of Davis et al.[2] for spin-stabilized rockets in the presence of all aerodynamic forces and moments afresh. In the earlier method considerable manipulation is required to arrive at the main condition utilizing the square root of a complex number. Such a criterion is found to present difficulty in obtaining useful analytic results for upper and lower bounds on stability.[3] Also, it is not possible to deduce the third condition, Eq. (19) of Ref. 2 as prescribed when d (as defined in Ref. 2) becomes zero, a situation which may arise in practice since the aerodynamic lift has the opposite effect to that of the Magnus moment. The two effects tend to approach each other and will become coincident at a certain stage of the motion leaving the Magnus force and the overturning moment to influence the projectile, which has been observed to be stable if spun fast enough.

The condition of Eq. (19) of Davis et al. has also been derived from the condition of Eq. (17) in a novel way, thus proving their contention (cf. p. 377, Ref. 2) that the latter condition is a consequence of the former.

Mathematical Preliminaries

Definition: Hurwitz polynomial–A polynominal

$$P(Z) = Z^n + (p_1 + iq_1)Z^{n-1} + \ldots + (p_n + iq_n) \tag{1}$$

is called a Hurwitz polynomial if all its zeros lie in the left half plane $Re(Z) < 0$.

Theorem.[4] The polynomial $P(Z)$ has all its zeros in the left half plane $Re(Z) < 0$ if and only if the determinants

$$\Delta_1 = p_1 \tag{2}$$

and

$$\Delta_k = \begin{vmatrix} p_1 & p_3 & p_5 & \cdots & p_{2k-1} & -q_2 & -q_4 & \cdots & -q_{2k-2} \\ 1 & p_2 & p_4 & \cdots & p_{2k-2} & -q_1 & -q_3 & \cdots & -q_{2k-3} \\ & & & \cdots & & & & \cdots & \\ 0 & & & \cdots & p_k & 0 & & \cdots & -q_{k-1} \\ 0 & q_2 & q_4 & \cdots & p_{2k-2} & p_1 & p_3 & \cdots & p_{2k-3} \\ 0 & q_2 & q_3 & \cdots & q_{2k-3} & 1 & p_2 & \cdots & p_{2k-4} \\ & & & \cdots & & & & \cdots & \\ 0 & & & \cdots & q_k & 0 & & \cdots & p_{k-1} \end{vmatrix} \tag{3}$$

for $k = 2, 3, \ldots, n$ $(p_2 = q_r = 0$ for $r > n)$ are all positive.

Derivation of the Stability Conditions

Utilizing the notations and assumptions of Davis et al., the characteristics equation of the spin stabilized rocket in the presence of all transverse aerodynamic forces and moments is governed by

$$Q(\lambda) = \lambda^2 + (b - ia)\lambda - (c + id) \tag{4}$$

where a, b, c, d are all constants.

We shall now find conditions that are necessary and sufficient for $Q(\lambda)$ to have both the roots in the left half plane. It may then be concluded that under those conditions $Q(\lambda)$ is a Hurwitz polynomial and stability of motion prevails.

A straightforward application of the preceding theorem to Eq. (4) yields

$$b > 0 \tag{5}$$

and

$$abd - b^2c - d^2 > 0 \tag{6}$$

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Inequalities (5) and (6), therefore, form a set of both necessary and sufficient conditions for the spin stabilized rocket to be stable,[5] in fact, asymptotically stable in the sense of Lyapunov.[6]

Condition (19) of Davis et al. may now easily be derived from the inequality

$$4(abd-b^2c-d^2)+(ab-2d)^2>0 \tag{7}$$

which reduces to

$$a^2-4c>0$$

on simplification even when $d=0$ holds true. Inequalities (5), (6), and (8) are essentially the stability conditions given by Nielsen and Synge.[7]

It may be mentioned that the method suggested can be applied when the coefficients in Eq. (4) are functions of some parameter, bounded, which is allowed to vary as in the case of motion with nonlinear aerodynamic forces and moments.

References

[1] Fuchs, B. A. and Levin, V. I., *Functions of a Complex Variable,* Pergamon Press, New York, 1961, Chap. V., Sec. 27, p. 251.

[2] Davis, L., Follin, J. W., and Blitzer, L., *Exterior Ballistics of Rockets,* D. Van Nostrand Company, Inc., Princeton, N. J., 1958, Chap. 10, Sec. 10.13, p. 374.

[3] Laitone, E. V., "Real Part of the Square Root of a Complex Number," *Journal of the Aeronautical Sciences*, Vol. 24, May 1957, pp. 391-392.

[4] Frank, E., "On the Zeros of Polynomials with Complex Coefficients," *Bulletin of the American Mathematical Society,* Vol. 52, Feb. 1946, pp. 144-157.

[5] Leipholz, H., *Stability Theory*, Academic Press, New York, 1970, Part I, Sec. 1.3, p. 24.

[6] Gantmacher, F. R., *The Theory of Matrices,* Vol. II, Chelsea Publishing Co., New York, 1959, Chap. XV, Sec. 5, p. 190.

[7] Nielsen, K. L. and Synge, J. L., "On the Motion of a Spinning Shell," *Quarterly of Applied Mathematics*, Vol. IV, No. 3, Oct. 1946, pp. 201-226.

Supporting Wire Interference Effects in Supersonic Near Wakes of Slender Bodies

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Introduction

HIGH-SPEED vehicles entering the atmosphere experience wake phenomena, which have an adverse effect on communications between the vehicle and some other transmitter and/or receiver located beyond the wake. Because of this communication problem, much investigation has been conducted to determine what exactly happens in the near and far wake.

One of the major problems encountered while investigating the near wake region of slender bodies, is that of supporting the model. Since the near wake region begins immediately behind the model and sometimes extends as far downstream as $X/H=4$, where X is the distance downstream of the base and H is the base height, the presence of any size sting will affect the base flow, the base pressure, and base heating. Side-mounted or wire-supported models are usually relied upon in order to obtain minimum disturbance measurements in the base region, with side-mounted systems being used primarily with two-dimensional models.

Due to the interference effects encountered when side-mounted systems are used for three-dimensional models, some investigators have turned to wire as a convenient method of model support. Considerable controversy has arisen in the past with regard to the amount of influence the wire supports have on the model flow patterns. The purpose of this paper is to investigate the effects support wires of different diameters have on the near viscous wakes of a wedge and a cone with the same included angle and base height or diameter.

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Literature Review

The degree of interaction between the support wires and model flowfield, as investigated by previous authors,[1-6] has resulted in considerable disagreement. The majority of this disagreement is concerned with: how many body diameters downstream the wire effects are observed; whether the disturbances are confined to the plane of the wire; and whether the model's wake neck shifts position because of the presence of the support wires. Mirly and Selberg[3] indicated that in the viscous wake, the presence of support wires has no effect on the pitot pressure for ratios of wire to model base diameters equal to or less than 0.007. Schmidt and Cresci[5] found no induced disturbances beyond 250 wire diam, while Chapkis and Garnage[7] measured disturbance past 1500 wire diam downstream. Mirly and Selberg[3] reported that in the nonviscous flow region, the effect of pitot pressure was lowered because of an interference wire, and this effect was not confined to the plane of the wire. These findings were in agreement with Dayman,[1] who observed shock waves induced by the wire in and outside the plane of the wire, whereas Chapkis and Garnage,[7] along with Hromas,[2] reported that the effect of wire is felt mainly in the plane of the wire support and that useful data can be obtained out of the plane of the wires. Dayman[1] stated that for M(Mach Number) ≤ 2, the support wires had negligible effect on the wake, whereas for $M>2$ the wake neck moved towards the base of the body. Ragsdale and Darling[4] observed that only for large wire to base diameter ratios did the wake neck move towards the base. Mirly and Selberg[3] concluded that in the near wake, the support wire did not interfere with wake growth and shock wave position. Hromas[2] concluded that the wire support appeared to have a significant effect only on the pitot pressures and not on the static pressures, indicating that the wire support does not induce a system of shock waves in the plane of the wire, but rather creates a quasi-steady, rather complicated vortex pattern in the plane of the wire. Pierce and Beecham[8] have reported that for wires at large angles of attack there is a periodic shedding of the boundary layer, which results in vortices that shed alternately from either side of the wire. In view of these apparent disagreements about wire induced effects, it was felt that more data, were needed to resolve that exact effects of wire supports.

Test Apparatus and Experimental Techniques

Wind Tunnel

All experiments were conducted in the University of Missouri-Rolla supersonic axisymmetric wind tunnel, which is an enclosed, free-jet, intermittent flow facility with a Mach 3.15 nozzle, at a Reynold's number of 2.14×10^6 per in. The tests were conducted at an operating stagnation pressure of 140 psig and at an average stagnation temperature of 500°R.